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Original article

Modeling diameter structures with the Log- Logistic function in natural forests of Durango, Mexico

Modelación de estructuras diamétricas con la función Log-Logistic en bosques naturales de Durango, México

Modelação de estruturas de diâmetro com a função Log-Logística na floresta natural de Durango, México

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SUMMARY

Diameter distribution models are useful tools for predicting forest growth and yield and planning sustainable forest management activities. The objective of this study was to analyze the fitting capacity of the Log- Logistic probability density function (PDF) using a percentile-based parameter estimator and to evaluate the accuracy of two alternative modeling approaches for diameter distributions in natural stands in northwestern Durango, Mexico. Six percentile estimators were evaluated and compared with the maximum likelihood method based on the performance of the Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and Cramér-Von Mises (W^2) statistics. For modeling diameter distributions, the graphical and numerical behavior of the prediction (PPM) and parameter recovery (PRM) methods were assessed with mean bias (SM) and mean absolute error (EMA). The best parameter estimator was the diameter that accumulates the 25th and 79% percentiles, considering the percentage of stands where it was most accurate in terms of KS, AD and W^2, as well as its performance with respect to maximum likelihood. Modeling the number of trees per diameter class with the PRM and PPM approaches resulted in similar accuracy based on the measurement of the square mean diameter, basal area per hectare, height and dominant diameter. This work contributes significantly by providing an easily applicable tool in growth models developed for the natural forests of the Sierra Madre Occidental in Mexico.

Keywords: mixed forests, diameter distribution, Log- Logistic function, percentiles, parameter prediction and recovery.

RESUMEN

Los modelos de distribuciones diamétricas son herramientas útiles para predecir el crecimiento y rendimiento de masas forestales y planear actividades de manejo forestal sustentable. El objetivo de este trabajo fue analizar la capacidad de ajuste de la función de densidad de probabilidad Log-Logistic a través de un estimador de parámetros basado en percentiles y evaluar la precisión de dos alternativas de modelización de las distribuciones diamétricas de rodales naturales del noroeste del estado de Durango. México. Se evaluaron seis estimadores de percentiles y se compararon con el método de máxima verosimilitud a partir del desempeño de los estadísticos Kolmogorov-Smirnov (KS), Anderson-Darling (AD) y Cramér-Von Mises (W^2). Para la modelización de la distribución diamétrica se evaluó el comportamiento gráfico y numérico de los métodos de predicción (PPM) y recuperación de parámetros (PRM) con el sesgo medio (SM) y error medio absoluto (EMA). El mejor estimador de parámetros resultó del diámetro que acumula los percentiles 25 y 79%, considerando el porcentaje de rodales donde fue más preciso en términos de KS, AD y W^2, así como su rendimiento respecto a máxima verosimilitud. La modelización del número de árboles por clase de diámetro con los enfoques PRM y PPM resultó tener similar precisión a partir de la medición del diámetro medio cuadrático, área basal por hectárea, altura y diámetro dominante. Este trabajo contribuye significativamente proporcionando una herramienta de fácil aplicación en los modelos de crecimiento desarrollados para los bosques naturales de la Sierra Madre Occidental en México.

Palabras clave: bosques mixtos, distribución diamétrica, función Log-Logistic, percentiles, predicción y recuperación de parámetros.

RESUMO

Os modelos de distribuição de diâmetros são ferramentas úteis para prever o crescimento e rendimento da massa florestal e planear atividades de gestão florestal sustentável. O objetivo deste trabalho foi analisar a capacidade de ajuste da função densidade de probabilidade Log-Logística através de um estimador de parâmetros baseado em percentis e avaliar a precisão de duas alternativas de modelagem das distribuições diamétricas de povoamentos naturais no noroeste do estado de Durango. México. Seis estimadores de percentis foram avaliados e comparados com o método de máxima verossimilhança com base no desempenho dos estatísticos de Kolmogorov-Smirnov (KS), Anderson-Darling (AD) e Cramér-Von Mises (W2). Para a modelagem da distribuição diamétrica, avaliou-se o comportamento gráfico e numérico dos métodos de predição (PPM) e recuperação de parâmetros (PRM) com o viés médio (SM) e o erro absoluto médio (EMA). O melhor estimador de parâmetros resultou do diâmetro que acumula os percentis 25 e 79, considerando a percentagem de povoamentos onde foi mais preciso em termos de KS, AD e W2, bem como o seu desempenho em relação à probabilidade máxima. A modelagem do número de árvores por classe de diâmetro com as abordagens PRM e PPM acabou por ter precisão semelhante a partir da medição do diâmetro quadrado médio, área basal por hectare, altura e diâmetro dominante. Este trabalho contribui significativamente ao fornecer uma ferramenta fácil de aplicar nos modelos de crescimento desenvolvidos para as florestas naturais da Sierra Madre Occidental no México.

Palavras-chave: florestas mistas, distribuição diamétrica, função Log-Logística, percentis, predição e recuperação de parâmetros.

INTRODUCTION

Mexico is among the ten countries with the largest area of natural forests and records all known vegetation types (National Forestry Commission [CONAFOR], 2013), which requires sustainable forest management to ensure the persistence of ecological functions, such as: carbon sequestration (which mitigates the greenhouse effect), microclimate regulation, and the protection of hydrological basins. Furthermore, the country's timber production is around 8.3 million cubic meters of forest roll ($m^3 r$) per year (Secretaría de Medio Ambiente y Recursos Naturales [SEMARNAT], 2018), with the states of Durango and Chihuahua accounting for 50% of national forest production. An important characteristic of Mexican forests is that more than 90% of the area belongs to the social sector, that is, to ejidos (common lands) and indigenous communities, making community forest management a particularly interesting prospect in terms of conservation and production.

Thus, to ensure the persistence of this resource, it is necessary to generate easy-to-implement tools through mathematical models of growth and yield that include state (stand) variables, such as: number of trees, basal area, height and dominant diameter and the frequency distribution by diameter class and surface unit. Knowledge of the number of trees per diameter class of the stand together with other variables at the individual tree level (i.e. volume, biomass) constitute an important tool for decision-making in community forest management (Nanos and Sjöstedt de Luna, 2017), and is one of the main parameters to quantitatively characterize the stand (Quiñones-Barraza *et al.*, 2015). In this sense, to describe the number of trees per unit area and diameter classes, the use of appropriate probability density functions (PDFs) with appropriate parameter estimation methods is required, such as: Beta, Gamma, Johnson's SB, Weibull and more recently the Burr XII and Log- Logistic functions. The latter have been used successfully with methods to estimate their parameters derived from moments, percentiles and maximum likelihood, with little emphasis on the search for more simplified parameter estimation procedures (Gorgoso-Varela, Ogana, & Ige, 2020; Ogana, 2020) without losing precision in the estimates.

The purpose of describing the number of trees per diameter class per unit area is to accurately estimate the PDF parameters that characterize the diameter distribution of both even-aged stands and those with irregular structures and different species mixes. This is achieved by relating the PDF parameters to stand variables whose future value can be obtained directly from a production table, a growth model, or another known variable. This stage is known as diameter distribution modeling (Corral-Rivas *et al.*, 2015). In this way, using stand models that allow estimating these variables for a future time, a diameter distribution can be reconstructed. There are two different methodologies to do this: the direct parameter prediction method (PPM) and the indirect parameter recovery method (PRM). Although research on the use of PDFs in the forestry field has spanned several decades, there are, however, some simplified and easy-to-use statistical distributions in community forest management, such as the Log- Logistic (LL) PDF, which has not been sufficiently studied in natural forests (Ogana and Dau, 2019; Ogana, 2020). The two-parameter LL PDF is a continuous probability distribution characterized by a shape parameter (alpha) and a scale parameter (beta), which represents an important advantage due to its simplicity and parsimony without the use of complex iterative procedures in parameter estimation (Ogana, 2020). Finding an alternative to estimate the parameters of the LL PDF has become a research topic to develop a simplified and accurate parameter estimator with the aim of reconstructing the diameter structures of all possible shapes that can be found in natural stands as an easy-to-use tool in community forest management.

Thus, the objective of this study was to analyze the fit capacity of the LL PDF by searching for a percentile-based parameter estimator and to evaluate the accuracy of two alternatives for modeling diameter distributions of natural forests in the northwest of the state of Durango, Mexico.

MATERIALS AND METHODS

Study area

The study area included natural stands (composed of multiple species with even-aged and inequalities) of the forests of the Piélagos Community, located in the municipality of Otáez , northwest of the state of Durango, Mexico. The area covers an area of 17,468.29 ha and is located between the geographic coordinates 105° 52' 22.36" W, 24° 45' 23.496" N. The average altitude above sea level is 2,521 m. The predominant climate is temperate, semi-cold, with long cool summers, sub-humid with summer rainfall greater than 10.2% annually (García, 2014). Precipitation is scarce, although intense at certain times of the year, with an average annual precipitation and temperature of 969.7 mm and 11.6 °C, respectively (García, 2014). The main vegetation groups are: pine forest, pine-oak forest, secondary shrub vegetation associated with pine forest and induced grassland (National Institute of Statistics and Geography [INEGI], 2017) (Figure 1).

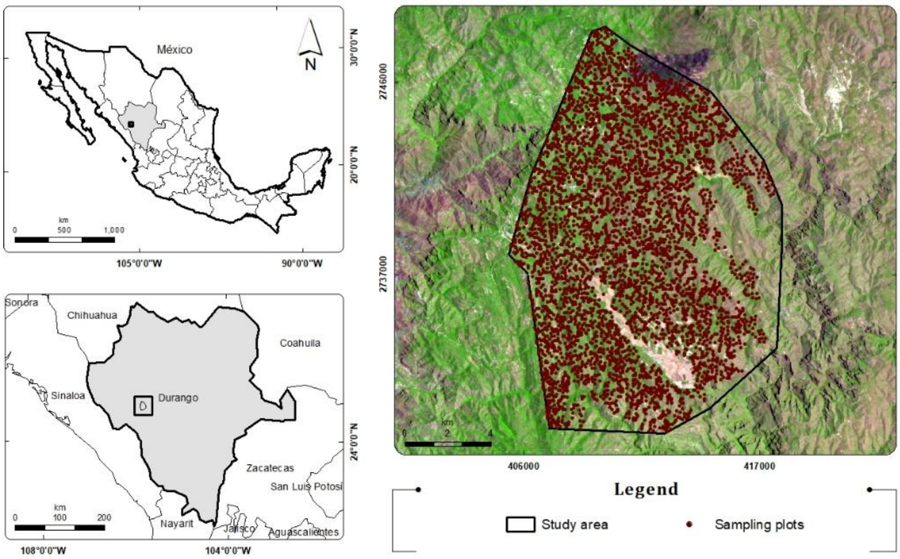

***Figure 1Location** of the study area*

Data

The database was obtained from 4,433 temporary sampling sites distributed across 748 management units or strata (stands), collected in 2015 as part of the timber forest management inventory of the Piélagos Community. The delimitation of these strata is defined by permanent characteristics such as land use, slope, exposure, and channels, allowing for the monitoring of dasometric variables in the different felling cycles over time, with the probability of change due to land use, the applied silvicultural system, or other variables (Meléndez-Soto, 2017). A stratified random sampling design without replacement was used, considering management units as strata, and the shape of the sites was circular, measuring 0.1 ha, where all trees with a normal diameter greater than 7.5 cm were measured.

The following conifer species were identified: white pine (Pinus *Arizona* Engelm); P. ayacahuite (P. *strobiformis* K. Ehrenb . ex-Schlecht); P. alazán (P. *durangensis* Martínez); P. real (P. *engelmannii* Carr); *P. herrerae* Martínez; p. Chinese (P. *leiophylla* Schlecht. and Cham); p. triste (P. *lumholtzii* Rob. and Fern); p. ocote (P. *teocote* Schlecht. And Cham.) and táscate (Juniperus *deppeana* Steud). In the case of broadleaf trees, the following species were identified: capulin (Prunus *serotonin* Ehrh); alder (Alnus *acuminata* Kunth); strawberry tree (Arbutus *xalapensis* Kunth); oaks (Quercus *conzattii* W. Trelease); (Q. *crassifolia* Humb. & Bonpl); (Q. *durifolia* Seemen ex Loes); (*Q. eduardii* W. Trelease); (*Q. obtusata* Bonpl); (Q. *rugosa* Née) and (Q. *sideroxyla* Humb. & Bonpl) (National Institute of Statistics and Geography [INEGI]. 2017)

The values of the descriptive statistics of the database and of the main variables of the stand are presented in Table 1.

Table 1. *- Summary of the database used in the adjustment and modeling of diameter distributions (sd = standard deviation)*

Variable	Mean ± (sd)	Range
Trees		
Normal diameter (*d*, cm)	21.9 ± (2.9)	7.5 – 80.0
Total height (*h*, m)	9.8 ± (1.6)	1.3 – 33.0
Asymmetry coefficient of d (*sk*, cm)	1.0 ± (0.4)	-0.1 - 2.5
Kurtosis coefficient of d (*kr*, cm)	3.7 ± (1.5)	1.6 - 13.5
Stands		
Dominant height (H_0, m)	14.1 ± (2.4)	8.8 - 26.4
Dominant diameter (D_0, m)	33.6 ± (4.9)	23.2 - 50.6
Square mean diameter (D_g)	24.0 ± (3.1)	17.0 - 38.4

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Basal area per hectare (G, m ²)	14.5 ± (4.8)	6.2 - 33.7
Number of trees per hectare (*N*)	316.7 ± (65.5)	156.0 - 596.7

Models

Logistic probability distribution function

Logistic PDF is a continuous probability distribution for a non-negative random variable (x). The probability density function f(x)and the cumulative distribution function F(x)are expressed by equations 1 and 2, respectively (Ogana , 2020) .

$$f(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\left[1+\left(\frac{x}{\beta}\right)^{\alpha}\right]^{-2} [1]$$

$$F(x)=\left[1+\left(\frac{\beta}{x}\right)^{\alpha}\right]^{-1} [2]$$

Where: αy β= parameters, shape and scale, respectively (αy β> 0), x = random variable (normal diameter).

Expression 2 has a closed form to facilitate the estimation of the proportion of the number of trees belonging to the different diameter classes without resorting to numerical integration (Ogana , 2020).

Parameter estimation with percentiles

The idea of the percentile-based parameter estimator for equation 2 arises from the principle developed by Clutter *et al*. (1983) for the Weibull PDF. If two sample percentiles are known, and each can be related to its cumulative distribution, the resulting equations are solved iteratively to obtain the values of the shape (α) and scale (β) parameters, respectively. Then, let be $X_p$the value of the percentile pin the sample, such that the percentile $X_p$is less than the 100% percentile. Therefore, the value of the percentiles is solved using equations 3 and 4.

$$P=\left[1+\left(\frac{\beta}{X_p}\right)^{\alpha}\right]^{-1} [3]$$

Derived from equation 3 the value of $X_p$is solved with equation 4.

$$X_p=\beta\left(\frac{1}{P}-1\right)^{-1/\alpha} [4]$$

The best parameter estimator based on sample percentiles was chosen from the following combinations: (M1) 25th and 79% percentiles; (M2) 17th and 97% percentiles; (M3) 24th and 93% percentiles; (M4) 40th and 80% percentiles; (M5) 30th and 70% percentiles; and (M6) 33rd and 67% percentiles.

Equation 4 for the different percentile combinations (M1 - M6) was solved with an iterative Newton-Raphson algorithm from the ***' lmfor ' package*** (Mehtätalo, 2022) implemented in an R code (R Core Team , (2021).

Estimation of parameters with maximum likelihood

To assess the suitability of the best parameter estimator based on percentiles of the distribution, it was compared with the maximum likelihood method (MLE, *Maximum Likelihood Estimation)*. According to Ogana (2020) MLE involves taking the partial derivative of the log-likelihood function of the Log- Logistic PDF with respect to the parameters (α and, βrespectively) and setting the equality expression to zero. The resulting function is solved using a numerical algorithm to obtain the parameter values iteratively. The log-likelihood function (θ) is expressed by equation 5 (Ogana, 2020):

$$\log(\theta) = n\cdot\log(\alpha)-n\cdot\log(\beta)+(\alpha-1)\sum_{i=1}^{n}\log(x_i) - n\cdot(\beta-1)\log(\beta) - 2\sum_{i=1}^{n}\left[1+\left(\frac{x_i}{\beta}\right)^{\alpha}\right] [5]$$

The partial derivative (∂) of equation 5 with respect to $\alpha$and βwill give equations 6 and 7, respectively:

$$\frac{\partial\theta}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\log(x_i) - n\cdot\log(\beta) - 2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\beta}\right)^{\alpha}\log\left(\frac{x_i}{\beta}\right)}{1+\left(\frac{x_i}{\beta}\right)^{\alpha}}=0 [6]$$

$$\frac{\partial\theta}{\partial\beta}=-\frac{n}{\beta}-\frac{n(\alpha-1)}{\beta}+\frac{2\alpha}{\beta}\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\beta}\right)^{\alpha}}{1+\left(\frac{x_i}{\beta}\right)^{\alpha}}=0 [7]$$

Since there are no explicit solutions for equations 6 and 7, estimates were obtained numerically using the " ***mledist " function from the " fitdistrplus "*** package (Dalignette and Dutang , 2015) implemented in R (R Core Team, 2021).

Statistics to assess goodness of fit

Three goodness-of-fit tests were used to assess the suitability of the parameter estimator based on percentiles. For each method, the Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and Cramer- Von Mises (W^2) statistics were estimated using equations 8, 9, and 10:

$$KS = \max|F_o(x_i) - F_e(x_i)|; 1 \leq i \leq n \text{ [8]}$$

$$AD = -n_i - \sum_{j=1}^{n_i}(2j-1)\left[\ln\left(F_e(x_j)\right) + \ln(1-F_o(x_{i-1}))\right]/n_i \text{ [9]}$$

$$W^2 = \sum_{i=1}^{n}\left\{F_e(x_i) - \frac{(i-0.5)}{n}\right\}^2 + \frac{1}{12n} \text{ [10]}$$

Where: $F_o(x_i)$= observed cumulative frequency distribution, $F_e(x_j)$= probability of the theoretical cumulative frequency of the diameter class (j), x_i= i- th observed value (values previously ordered from lowest to highest), n_i= number of trees (i=1,…, n), max = maximum difference, ln= natural logarithm.

Each fitting method has different strengths and weaknesses, leading to a diversity of values in the goodness-of-fit statistics (tests), therefore, to select the best parameter estimator the relative rank index (R_i) introduced by Poudel and Cao (2013) and recently used by Sun was used. *et al.* (2019) and Ogana (2020) equation 11:

$$R_i = \sum_k^3\left[1 + \frac{(m-1)(S_{i,k} - S_{min})}{S_{máx} - S_{min}}\right] \text{ [11]}$$

Where: m= total number of adjustment methods evaluated (m=7), $S_{i,k}$= value of the i statistic with the goodness-of-fit test k, $S_{min}$and $S_{máx}$= minimum and maximum value of, $S_k$respectively.

The value $R_i$must be a real number between 1 (best) and 7 (worst) for each method evaluated with the three goodness-of-fit tests; therefore, the value $R_i$resulted from the sum of $R_i$each estimator, including MLE. Finally, this value was rated with a real number from 1 to 7, with 1 being the $R_i$lowest value in each stand and 7 the highest.

The accuracy of the LL PDF fit with parameters estimated from percentiles and MLE was based on residual plots and numerical comparisons of the following statistics: (1) mean bias (SM) and (2) mean absolute error (MEE). The expressions are given in Eqs. 12 and 13:

$$\mathbf{SM}=\frac{\sum_{i=1}^{n}(F_o(x_i)-F_e(x_i))}{n} \quad [12]$$

$$\mathbf{EMA}=\frac{\sum_{i=1}^{n}|F_o(x_i)-F_e(x_i)|}{n} \quad [13]$$

Where: $F_o(x_i), F_e(x_i)$= the observed (actual value) and theoretical (estimated) frequencies, respectively. n= Number of observations.

Modeling of diameter distribution

One of the objectives of this study is to obtain the parameters of the LL PDF when there is no diameter inventory, but only stand variables. Following the PPM approach, the parameters estimated with the best percentile estimator were related to the main stand variables using a multiple linear regression model with the following form (equation 14):

$$Y_i=f(X_i,\beta_i)+\varepsilon_i \quad [14]$$

Where: Y_i= vector of the parameters and percentiles of the distribution, X_i= vector of the predictor variables (stand variables), β_i= parameters associated with the variable X_i, ε_i= error term.

On the other hand, the PRM approach consists of recovering the parameters of the LL PDF estimated from the best percentile estimator, considering that the percentile values of the diameter distribution are easy to model dynamically given their relationship with stand variables (Hyink and Moser , 1983) , in a first stage these percentiles were linearly related to stand variables from equation 14 and, subsequently in a second stage the parameters were estimated from the fundamental relationships that estimate the percentiles of the distribution.

Finally, the fit analysis of allometric relationships (parameters and percentiles with stand variables) was based on residual plots and numerical comparisons of the root mean square error (RMSE), coefficient of determination values (R^2), corrected Akaike information criterion (AIC_c), and Schwarz's Bayesian information criterion (BIC) that compare the equations in terms of their parsimony (simplicity). The expression of the statistics is represented in the following equations 15, 16, 17 and 18.

$$RMSE=\sqrt{\frac{\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2}{n-p}} \quad [15]$$

$$R^2=1-\frac{(n-1)\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2}{(n-p)\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2} \quad [16]$$

$$AIC_c=n\ln\left(\frac{SSR}{n}\right)+2p \quad [17]$$

$$BIC=n\ln\left(\frac{SSR}{n}\right)+p\ln(n) \quad [18]$$

Where: y_i= observed values, $\hat{y}_i$= predicted values, $\bar{y}$= average, n= total number of observations, p= number of parameters in the equation, SSR= sum of the squares of the residuals, ln= natural logarithm.

RESULTS AND DISCUSSION

Adjustment of the FDP Log- Logistic

The evaluation of the methods to estimate the parameters of the Log- Logistic PDF using the relative rank criterion with the three goodness-of-fit tests (KS, ADand) W^2resulted in M1 being the best with 30.7% of the stands evaluated. This percentage was very similar to the fit with MLE (32.8%), indicating its suitability for estimating the parameters from the 25th and 79th percentiles of the diameter distribution. In contrast, the lowest percentages were presented in ascending order in M2, M4, M3, M6 and M5 (1.9, 8.2, 8.2, 8.6 and 9.8%, respectively), because they recorded the highest relative range values (Figure 2).

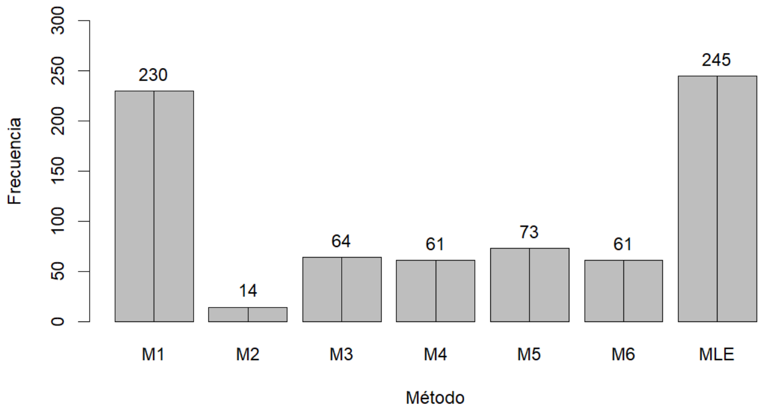

Figure 2. - *Number of stands with the lowest relative rank value for the different Log -Logistic PDF parameter estimation methods*

The average values of the parameters and statistics of the goodness-of-fit tests that turned out to have the lowest value of the relative range are presented in Table 2.

Table 2. *- Fit statistics for different variables used as predictor variables to model the maximum crown diameter of Pinus Cooperi*

Estimator		Estimator with percentiles						MLE
		M1	M2	M3	M4	M5	M6	
Parameter	Alpha	3.43	4.23	3.77	3.43	3.44	3.46	4.02
	Beta	19.62	18.64	18.93	19.37	19.55	19.56	19.71
Goodness-of-fit test	KS	0.14	0.18	0.15	0.15	0.14	0.15	0.12
	AD	4.49	8.27	5.41	6.09	5.25	5.94	3.38
	W^2	0.65	1.36	0.89	0.80	0.67	0.74	0.50

M1: 25th and 79% percentiles, M2: 17th and 97% percentiles, M3: 24th and 93% percentiles, M4: 40th and 80% percentiles, M5: 30th and 70% percentiles, M6: 33rd and 67% percentiles, MLE: maximum likelihood estimation, KS: Kolmogorov-Smirnov, AD: Anderson-Darling, W^2*: Cramer- Von Mises.*

When comparing the mean parameter values (Table 2), all percentile methods evaluated generally yielded very similar values to those estimated with MLE. Meanwhile, the mean values of the goodness-of-fit test statistics were higher with the percentile estimators than with MLE. Since M1 proved to be the most efficient parameter estimator (higher percentage of stands with a lower relative range value), it was compared with MLE in subsequent analyses to evaluate its accuracy in estimating the number of trees per diameter class.

Furthermore, in order to find the existence of some type of systematic pattern, the behavior of the error (bias) in the prediction of the number of trees per diameter class of M1 versus MLE was studied graphically (Figure 3).

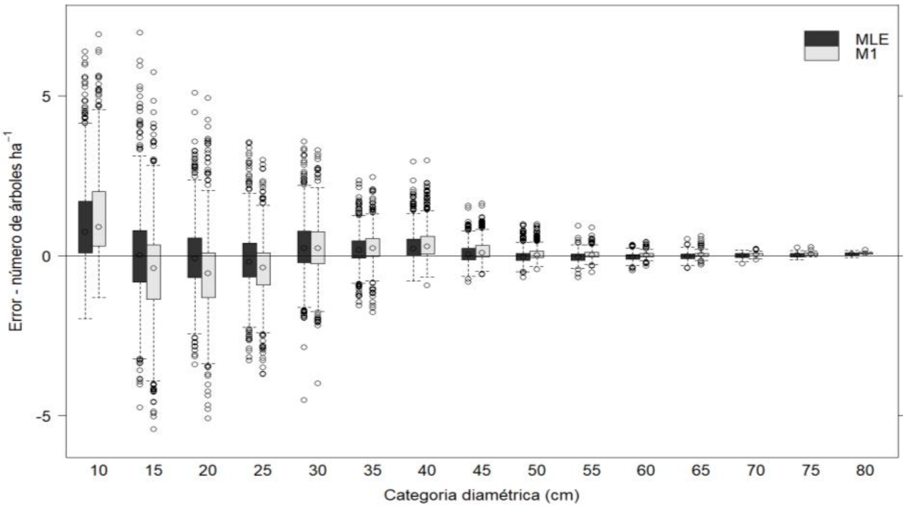

Figure *3- Error behavior in the prediction of the number of trees per diameter class with the Log- Logistic PDF with parameters estimated with* ***the*** *M1 percentile method (25 and 79%) and maximum likelihood (MLE)*

In view of Figure 3, the behavior of the mean errors by diameter class shows that both parameter estimation methods (MLE and M1) turn out to be very efficient (very similar errors around zero) from the diameter class of 10 cm, but not in smaller classes, where both methods slightly overestimate the number of trees with a very marginal improvement in precision with the MLE compared to M1.

In general, the performance of the M1 graphically and numerically evaluated with SMand EMAproved to be efficient, so it is considered an attractive option for use in modeling the diameter distributions of mixed masses in northwest Durango at any time interval due to its simplicity, therefore, the rest of the analyses are limited to its application.

The recovery of the parameters (PRM) of the Log- Logistic PDF was based on relating the percentiles $X_{25}$and $X_{79}$of the diameter distribution with stand variables and subsequently estimating these percentiles from functional relationships (regression functions). The diameter corresponding to 25% of the distribution showed a greater correlation (Pearson's correlation index significant at 5%) with the mean square diameter (D_g), followed by the basal area per hectare (G) and dominant diameter (D_0). Meanwhile, the diameter corresponding to 79% of the distribution turned out to have a greater (positive) relationship with the D_g, D_0, Gand dominant height (H_0).

Regarding the prediction of the parameters (PPM) of the Log- Logistic PDF , the parameter αthat describes the shape of the distribution presented a Pearson correlation index of -0.87 with the transformed variable Lq= $1/\ln(X_{25}/D_g)$y, low correlation, but significant (p > 0.05) with H_0y. D_0For its part, the parameter β(scale) presented a significant correlation (p > 0.05) with D_g, D_0, andG.

The resulting equations from the two approaches to modelling the diameter distribution are equations 19, 20, 21 and 22:

$$(RMSE = 1.38, R2 = 0.76, AICc = 488.2\ y\ BIC = 511.2)[19]$$

$$X_{79}=-10.555+0.889*D_g-0.534*G+0.742*D_0+0.121*H_0+\varepsilon_i[20]$$

$$(RMSE = 1.55, R2 = 0.88, AICc = 661.5\ y\ BIC = 689.1)$$

$$\alpha=2.852-1.019*L_q-0.031*H_0-1.012*D_0+\varepsilon_i$$

$$(RMSE = 0.389, R2 = 0.77, AICc = -1435\ y\ BIC = -1412)[21]$$

$$\beta=5.868+1.354*D_g-0.736*D_0+0.421*G+\varepsilon_i$$

$$(RMSE = 0.563, R2 = 0.96, AICc = -844\ y\ BIC = -821)[22]$$

Where: ε_i= error term.

It is interesting to confirm that all parameters of equations 19, 20, 21 and 22 turned out to be significantly different from zero with a significance level of 5% ($p < 0.05$).

Overall, when comparing both diameter distribution modeling methodologies (PPM and PRM) and the statistics used to assess their accuracy, they turned out to have very similar values. The average bias weighted by the number of trees per diameter class was 0.06, 0.22, and 0.07 for PPM, PRM, and MLE, respectively. The mean absolute error was 0.95 for both modeling methodologies (PPM and PRM) and 0.92 for MLE. To conduct a more detailed inspection of both modeling approaches, the error behavior by diameter class was graphically represented (Figure 4).

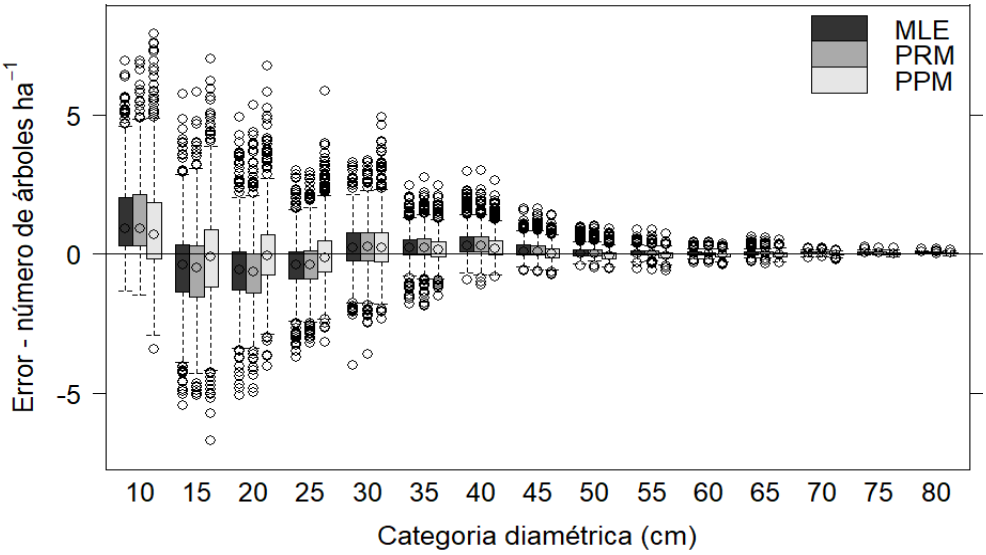

***Figure** 4- Bias behavior in the modeling of the diameter distribution with the parameter recovery and prediction approaches (PRM and PPM) versus the parameter estimation of the Log- **Logistic** PDF with MLE*

In Figure 4, the evolution of SMboth methodologies results in very similar deviations starting from the 30 cm diameter class; however, it is clearly observed that for classes smaller than 25 cm, the closest deviations around zero are achieved with the direct parameter prediction method (PPM). It is interesting to mention that most of the studied masses are represented by these diameter classes; therefore, this modeling methodology turned out to be marginally more accurate than the indirect parameter recovery method (PRM). Regarding the "EMA" behavior, both modeling approaches turned out to have very similar deviations across the entire range of diameter classes. In addition, it is observed that the absolute errors of both modeling methodologies are very similar to those obtained during parameter estimation with MLE, confirming their validity and accuracy.

Estimating the parameters of the Log- Logistic PDF using the percentile method offered simplicity and attractive results based on the statistics used to assess goodness of fit. Of the six methods evaluated and compared with MLE using the relative range, it was demonstrated that, starting from the 25th and 79th percentiles of the diameter distribution, the parameters to describe the diameter frequencies of the studied natural stands could be accurately estimated. Similar results were reported by Ogana. (2020) in two natural forests and two *Gmelina arbórea* plantations Roxb . and *Tectona grandis* Linn. f., in Nigeria, by fitting this distribution to the 40th and 80th percentiles of the diameter distribution. In addition, Wang and Rennolls (2005) demonstrated that this distribution was efficient with respect to the Weibull, Johnson SB, Beta and Burr VII PDFs by estimating parameters by MLE in Chinese Fir plantations.

Regarding the accuracy of the percentile estimator (M1), it was only slightly surpassed by MLE, covering 63.5% of the stands with both parameter estimation methods, which validates the use of the Log- Logistic PDF in this type of forest stands. Pogoda, *et al.*, (2019) demonstrated the efficiency of using percentiles to estimate Weibull PDF parameters versus the method of moments in 163 stands of *Alnus glutinosa* (L.) Gaertn . In contrast, studies such as George and Ramachandran (2011); Quiñonez-Barraza *et al.* (2015); Corral-Rivas *et al.* (2015); Gorgoso -Varela *et al.* (2020) demonstrate that the use of percentiles to fit different PDFs is not efficient. In this context, Wang et al. (2005), mention that differences in the parameter estimation method, in the use of probability distribution functions, statistics to measure goodness of fit and, in the data, set used, contribute to different conclusions, therefore, there is no reason why a single PDF and method of estimating its parameters is the most appropriate to describe diameter frequencies of a specific forest stand.

In this work, the efficiency of the percentile estimator was demonstrated by comparing it with MLE, which requires complex iterative processes based on three goodness-of-fit statistics. In this regard, Ogana (2020) emphasizes that, unlike the MLE method and ordinary least squares regression, the percentile estimator is more simplified and practical and does not require a complex iterative procedure. Corral-Rivas *et al.* (2015)

establish that the parsimony of a PDF is a desirable characteristic of empirical models, that is, the simpler the parameter estimation, the greater its potential for practical applications.

Regarding the modeling of diameter distribution at any stage of stand development, previous studies that have analyzed the accuracy of parameter prediction (PPM) and parameter recovery (PRM) methods differ in results. While some emphasize the predictive capacity of the indirect PRM estimation method, others emphasize the accuracy and parsimony of the direct PPM prediction method. Jiang and Brooks (2009), who studied both methods in *Pinus plantations palustris* Mill. showed that the direct estimation method PPM is more accurate, partly coinciding with the results found in this study. While Cao (2004), tested six methods to predict the parameters of the Weibull PDF, among them he evaluated the indirect method PRM, finding good results with the latter. Also, Leduc *et al.* (2001), found similar results to this study when comparing the prediction and recovery method of Weibull PDF parameters in pine plantations in the southern United States. Likewise, Gorgoso -Varela *et al*. (2007) found that the indirect parameter recovery method is more accurate in modeling diameter structures of *Betula alba* L., in northwestern Spain, with the two-parameter Weibull PDF.

Regarding the PRM, the variables that had the greatest influence in estimating the accumulated diameter at the 25th and 79th percentiles were the quadratic mean diameter, dominant diameter, and basal area per hectare. For predicting the shape parameters (α) and scale (β), the stand variables that had the greatest relationship were the height and dominant diameter, quadratic mean diameter, and basal area per hectare. Ogana (2020) found these same relationships studying two natural forests and two *Gmelina arbórea* plantations. Roxb . and *Tectona grandis* Linn. f., in Nigeria, concluding that both parameter estimation methods are efficient and reliable for estimating Log-Logistic PDF parameters. However, Quiñonez-Barraza *et al.* (2015) recommends PPM to characterize diameter distributions of mixed stands of *Pinus* and *Quercus genera* in northwestern Durango, Mexico, since Weibull PDF parameters can be predicted with easily measured stand variables (dominant height, basal area, square mean diameter). In this regard, Torres-Rojo *et al*. (2000) presented a strategy to improve the accuracy of parameter estimation with PPM using the Weibull PDF in central Mexico.

Overall, in this work, PPM can be considered the best option for modeling diameter structures, given that it proved to be marginally more accurate in the diameter classes where they are most common in the studied stands (10–35 cm). Furthermore, this is due to the fact that the variables that estimate the parameters are easy to measure during conventional management inventories.

CONCLUSIONS

Modeling the diameter distribution of natural forests studied using the PPM and PRM methods of the Log- Logistic PDF proved to be efficient in estimating the number of trees per diameter class using easily measured stand variables (Dg, G, H0, and D0). The best parameter estimators were the 25th and 79th percentiles, considering the relative range and its performance compared to parameter estimation using the maximum likelihood method. Therefore, it is marginally demonstrated that PPM was more efficient in reconstructing the diameter distribution of the stand at any age. These results contribute significantly to providing an easily applicable tool for community forest management of natural forests in northwestern Durango, Mexico.

Finally, it is recommended to use another FDP to describe and model those forest stands that were not accurately covered with Log- Logistic FDP, considering the goodness-of-fit statistics analyzed.

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REFERENCES

CAO, Q.V., 2004. Predicting Parameters of a Weibull Function for Modeling Diameter Distribution. *ForestScience* [En línea]. Vol. 50, n.° 5, pp. 682–685 [Consulta: 21 de noviembre de 2024]. ISSN 1938-3738. DOI: 10.1093/forestscience/50.5.682. Disponible en: https://academic.oup.com/forestscience/article/50/5/682/4617271

CLUTTER, J.L., FORTSON, J.C., PIENAAR, L.V., BRISTER, G.H., y BAILEY, R.L., 1983. *Timber Management: A Quantitative Approach*. New York, John Wiley & Sons. pp. 333. ISBN 0471029610.

COMISIÓN NACIONAL FORESTAL (CONAFOR), 2013. Bosques, cambio climático y REDD+ en México. Guía básica (segunda edición) [En línea]. [Consulta: 21 de noviembre de 2024]. Disponible en: http://www.conafor.gob.mx:8080/documentos/docs/35/4034Gu%C3%ADa%20B%C3%A1sica%20de%20Bosques,%20Cambio%20Clim%C3%A1tico%20y%20REDD_%20.pdf

CORRAL-RIVAS, S., ÁLVAREZ-GONZÁLEZ, J.G., CORRAL-RIVAS, J.J., y LÓPEZ-SÁNCHEZ, C.A., 2015. Characterization of diameter structures of natural forests of northwest of Durango, Mexico. *Revista Chapingo Serie Ciencias Forestales y del Ambiente* [En línea]. Vol. 21, n.° 2, pp. 221–236 [Consulta: 21 de noviembre de 2024]. ISSN 2007-4018. DOI: 10.5154/r.rchscfa.2014.10.046. Disponible en: https://revistas.chapingo.mx/forestale/last_issues/?section=articles&subsec=issues&numero=204&articulo=1993

DELIGNETTE M, M.L., y DUTANG, C., 2015. Fitdistrplus: An R Package for Fitting Distributions. *Journal of Statistical Software* [En línea]. Vol. 64, n.° 4, pp. 1-34 [Consulta: 21 de noviembre de 2024]. ISSN 1548-7660. DOI: 10.18637/jss.v064.i04. Disponible en: https://www.jstatsoft.org/article/view/v064i04

GARCÍA, E., 2014. *Modificaciones al sistema de clasificación climática Köppen*. Ciudad de México: IG, UNAM.

GEORGE, F., y RAMACHANDRAN, K.M., 2011. Estimation of parameters of Johnson's system of distributions. *Journal of Modern Applied Statistical Methods* [En línea]. Vol. 10, n.° 2, pp. 494-504 [Consulta: 21 de noviembre de 2024]. ISSN 1538-9472. DOI: 10.22237/jmasm/1320120480. Disponible en: https://jmasm.com/index.php/jmasm/article/view/559

GORGOSO-VARELA, J.J., ÁLVAREZ-GONZÁLEZ, J.G., ROJO-ALBORECA, A., y GRANDAS-ARIAS, J.A., 2007. Modelling diameter distributions of Betula alba L. stands in northwest Spain with the two-parameter Weibull function. *ForestSystems*[En línea]. Vol. 16, n.° 2, pp. 113-123. [Consulta: 02 de septiembre de 2024]. ISSN 2340-3578. DOI: 10.5424/srf/2007162-01002. Disponible en: https://fs.revistas.csic.es/index.php/fs/article/view/1002

GORGOSO-VARELA, J.J., OGANA, F.N., y IGE, P.O., 2020. A comparison between derivative and numerical optimization methods used for diameter distribution estimation. *Scandinavian Journal of Forest Research* [En línea]. Vol. 35, n.° 3-4, pp 156-164. [Consulta: 21 de julio de 2024]. ISSN 1651-1891. DOI: 10.1080/02827581.2020.1760343. Disponible: https://www.tandfonline.com/doi/abs/10.1080/02827581.2020.1760343

HYINK, D.M., y MOSER, J.W., 1983. A generalized framework for projecting forest yield and stand structure using diameter distributions. *ForestScience* [En línea]. Vol. 29, n.° 1, pp. 85-95. [Consulta: 25 de julio de 2024]. ISSN 1938-3738. DOI: 10.1093/forestscience/29.1.85 Disponible en: https://academic.oup.com/forestscience/article-abstract/29/1/85/4656683?login=false

INSTITUTO NACIONAL DE ESTADÍSTICA Y GEOGRAFÍA (INEGI), 2017. Guía para la interpretación de cartografía: Uso del Suelo y Vegetación. Escala 1:250,000. Serie VI. México [En línea]. [Consulta: 04 de septiembre de 2024]. Disponible en: http://internet.contenidos.inegi.org.mx/contenidos/Productos/prod_serv/contenidos/espanol/bvinegi/productos/nueva_estruc/702825092030.pdf

JIANG, L., y BROOKS, J. R., 2009. Predicting diameter distributions for young longleaf pine plantations in Southwest Georgia. *Southern Journal of Applied Forestry* [En línea]. Vol 33, n.° 1, pp. 25-28. [Consulta: 29 de agosto de 2024]. ISSN 1938-3754. DOI: 10.1093/sjaf/33.1.25. Disponible: https://colab.ws/articles/10.1093%2Fsjaf%2F33.1.25

LEDUC, D.J., MATNEY, T.G., BELLI, K.L., y BALDWIN, V.C., 2001. Predicting diameter distributions of longleaf pine plantations: A comparison between artificial neural networks and other accepted methodologies. Asheville, NC: U.S. Dept. of Agriculture, Forest Service, Southern Research Station. pp. 18.

MEHTÄTALO, L., 2017. Lmfor: functions for Forest Biometrics. R packageversion 1.2 [En línea]. [Consulta: 29 de julio de 2024]. Disponible en: https://cran.r-project.org/web/packages/lmfor/lmfor.pdf

MELÉNDEZ-SOTO, A., LÓPEZ-SÁNCHEZ, C. A., LÓPEZ-SERRANO, P. M., MONTIEL-ANTUNA, E., SOLÍS-MORENO, R., y CORRAL-RIVAS, J. J. 2017. Comparación de técnicas de rodalización en masas forestales de la Sierra Madre Occidental en el estado de Durango. Revista Mexicana de Ciencias Forestales 8 (41). México, ME. https://doi.org/10.29298/rmcf.v8i41.26

NANOS, N., y SJÖSTEDT DE LUNA, S.,2017. Fitting diameter distribution models to data from forest inventories with concentric plot design. *ForestSystems*[En línea]. Vol. 26, n.° 2, pp. e01S. [Consulta: 11 de septiembre de 2024]. ISSN 2171-9845. DOI: 10.5424/fs/2017262-10486. Disponible: https://fs.revistas.csic.es/index.php/fs/article/view/10486

OGANA, F. N., y DAU, J. H. 2019. Deriving Tree Crown Distributions from Diameter at Breast Height. *Journal of Tropical Forestry and Environment* [En línea]. Vol. 9, n.° 1, pp. 80-88. [Consulta: 03 de septiembre de 2024]. ISSN 2235-9362. DOI: 10.31357/jtfe.v9i1.3954. Disponible en: http://journals.sjp.ac.lk/index.php/JTFE/article/view/3954

OGANA, F.N., 2020. A percentile-based estimator for the Log-Logistic function: Application to forestry. *ForestryStudies* [En línea]. Vol. 72, n.° 1, pp. 107-120. [Consulta: 27 de agosto de 2024]. ISSN 1736-8723. DOI: 10.2478/fsmu-2020-0009. Disponible en: https://sciendo.com/article/10.2478/fsmu-2020-0009

POGODA, P., OCHAL, W., y ORZEL, S., 2019. Modeling diameter distribution of black alder (Alnusglutinosa (L.) Gaertn.) stands in Poland. *Forests* [En línea]. Vol. 10, n.° 5, pp. 412. [Consulta: 08 de agosto de 2024]. ISSN 1999-4907. DOI: 10.3390/f10050412. Disponible en: https://www.mdpi.com/1999-4907/10/5/412

POUDEL, K.P., y CAO, Q.V., 2013. Evaluation of methods to predict Weibull parameters for characterizing diameter distributions. *ForestScience*[En línea]. Vol. 59, n.° 2, pp. 243-252. [Consulta: 17 de septiembre de 2024]. ISSN 1938-3738. DOI: 10.5849/forsci.12-001. Disponible en: https://academic.oup.com/forestscience/article/59/2/243/4583693

QUIÑONEZ-BARRAZA, G., DE LOS SANTOS-POSADAS, H.M., CRUZ-COBOS, F., VELÁZQUEZ-MARTÍNEZ, A., ÁNGELES-PÉREZ, G., y RAMÍREZ-VALVERDE, G., 2015. Modelación dinámica de distribuciones diamétricas en masas mezcladas de Pinus en Durango. México. *Madera y Bosques* [En línea]. Vol. 21, n.° 2, pp. 59-71. [Consulta: 22 de julio de 2024]. ISSN 1405-0471. DOI: 10.21829/myb.2015.212445. Disponible en: https://myb.ojs.inecol.mx/index.php/myb/article/view/445

R CORE TEAM. 2021. *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. Disponible en: http://www.Rproject.org/

SECRETARÍA DE MEDIO AMBIENTE Y RECURSOS NATURALES (SEMARNAT), 2018. Anuario Estadístico de la Producción Forestal. Dirección General de Gestión Forestal y de Suelos. Estado de México [En línea]. [Consulta: 21 de agosto de 2024]. Disponible en: https://www.gob.mx/semarnat/documentos/anuarios-estadisticos-forestales

SUN, S., CAO, Q.V., y CAO, T., 2019. Characterizing diameter distributions for uneven-aged pine-Oak mixed forests in the Qinling Mountains of China. *Forests* [En línea]. Vol. 10, n.° 7, pp. 596. [Consulta: 17 de julio de 2024]. ISSN 1999-4907. DOI: 10.3390/f10070596. Disponible en: https://www.mdpi.com/1999-4907/10/7/596

TORRES-ROJO, J. M., MAGAÑA-TORRES, O. S., y ACOSTA-MIRELES, M. 2000. Metodología para mejorar la predicción de parámetros de distribuciones diamétricas. *Agrociencia*[En línea]. Vol. 34, n.° 5, pp. 627-637. [Consulta: 04 de septiembre de 2024]. ISSN: 1405-3195. Disponible en: http://www.redalyc.org/articulo.oa?id=30234512

WANG, M., &RENNOLLS, K. 2005. Tree diameter distribution modelling: introducing the Logit-Logistic distribution. *Canadian Journal of Forest Research* [En línea]. Vol. 35, n.° 6, pp. 1305-1313. [Consulta: 15 de agosto de 2024]. ISSN 1208-6037. DOI: 10.1139/x05-057. Disponible en: https://cdnsciencepub.com/doi/10.1139/x05-057

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