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Equation to predict the maximum crown diameter of *Pinus cooperi* Blanco in Durango, México

Ecuación para predecir el diámetro máximo de copa de Pinus cooperi Blanco en Durango, México

Equação para prever o diâmetro máximo da copa de Pinus cooperi Blanco em Durango, México

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ABSTRACT

The maximum crown area is an important variable in the estimation of the level of competition that affects the growth potential and productivity of species. Therefore, the objective of the present study was to adjust regression models to predict the maximum crown diameter for *Pinus cooperi* Blanco in the forest region of El Salto, in the state of Durango. In particular, 95 trees of different diameter categories and growing free of competition were used. Independently, three regression models were analyzed, simple linear, second-degree polynomial and potential, to predict the maximum canopy diameter as a function of normal diameter, total height, height at the base of the crown, age and total length of the crown. The results showed that the second-degree polynomial model with the inclusion of normal diameter as an independent variable presented the best fit in the prediction of the maximum crown diameter. In general, normal diameter was the most significant variable for the prediction of maximum crown diameter in *Pinus cooperi*. Based on the results, two forest productivity scenarios related to crown competition factor (CCF) can be identified; when the CCF is 100 % or higher, timber productivity is optimal and when it is less than 100 %, productivity is not optimal, which can be attributed to little or no competition and deficiency in land use.

Keywords: maximum area, competition, models, timber productivity

RESUMEN

El área máxima de copa es una variable importante en la estimación del nivel de competencia que afecta al potencial de crecimiento y productividad de las especies. Por lo tanto, el objetivo del presente estudio fue ajustar modelos de regresión para predecir el diámetro máximo de copa para *Pinus cooperi* Blanco en la región forestal de El Salto, en el estado de Durango. Particularmente se utilizaron 95 árboles de diferentes categorías diamétricas y creciendo libres de competencia. De manera independiente, se analizaron tres modelos de regresión, lineal simple, polinómico de segundo grado y potencial, para predecir el diámetro máximo de copa en función del diámetro normal, altura total, altura a

la base de la copa, edad y longitud total de la copa. Los resultados evidenciaron que el modelo polinómico de segundo grado con la inclusión del diámetro normal como variable independiente presentó el mejor ajuste en la predicción del diámetro máximo de copa. En general el diámetro normal fue la variable más significativa para la predicción del diámetro máximo de copa en *Pinus cooperi*. Con base a los resultados se pueden identificar dos escenarios de productividad del bosque relacionados con factor de competencia de copa (FCC), cuando este es del 100 % o superior, la productividad maderable es óptima y cuando es menor al 100% la productividad no es óptima pudiendo estar atribuido a la poca o nula competencia y deficiencia en el uso del suelo.

Palabras clave: área máxima, competencia, modelos, productividad maderable.

RESUMO

A área máxima do dossel é uma variável importante na estimativa do nível de competição que afeta o potencial de crescimento e produtividade das espécies. Portanto, o objetivo deste estudo foi ajustar modelos de regressão para prever o diâmetro máximo da copa de *Pinus cooperi* Blanco na região florestal de El Salto, no estado de Durango. Em particular, serão utilizadas 95 árvores de diferentes categorias de diâmetro, criando limites à concorrência. De forma independente, foram analisados três modelos de regressão, linear simples, polinomial de segundo grau e potencial, para determinação do diâmetro máximo da copa em função do diâmetro normal, altura total, altura na base da copa, idade e comprimento total da copa. a xícara Os resultados mostraram que o modelo polinomial de segundo grau com a inclusão do diâmetro normal como variável independente apresentou o melhor ajuste na predição do diâmetro máximo da coroa. De modo geral, o diâmetro normal foi a variável mais significativa para predizer o diâmetro máximo da copa em *Pinus cooperi*. A partir dos resultados podem ser identificados cenários de produtividade florestal relacionados ao FCC; Quando o FCC é 100% ou superior, a produtividade da produção é ótima e quando é inferior a 100%, a produtividade não é ótima e pode ser atribuída a esse tempo ou à falta de competição e à deficiência no uso da terra.

Palavras-chave: área máxima, concorrência, modelos, produtividade sustentável.

INTRODUCTION

Timber production depends on the level of productivity existing within a given forest area. In the period from 1990 to 2017, the total national timber production was, on average, 6.86 million m³ of roundwood, and pine contributed 79.38 % (SEMARNAT, 2019), but the effective use of the land and the quality of the forest products produced there depend on the management of the density of the stands throughout the rotation, in this sense, management can be considered as a fundamental quantitative tool for planning, execution and evaluation that defines silvicultural interventions (Santiago-García *et al* ., 2013; Vospernik and Sterba, 2015; Tamarit *et al*., 2020). In addition to the above, forest development depends on the different morphological attributes of the trees such as the shape of the trunks, roots and crowns, components that are usually evaluated to predict the growth and productivity of forests. However, there are few studies that consider crown parameters (Nájera and Hernández, 2008), despite the fact that the variables related to tree crowns offer interdimensional information such as the surface area occupied by an individual, level of competence and vitality (Hess *et al*., 2016; Cisneros *et al*. , 2019; Givnish, 2020). Therefore, forest management defines the morphometric relationships in forest communities, so their adequate description and characterization can help in the evaluation of silvicultural practices (Soto *et al*., 2016).

The size, structure and shape of tree canopies determine the extent and efficiency of physiological processes such as photosynthetic activity, transpiration and respiration which in turn determine the growth, development and productivity of the forest (Sharma *et al*., 2017; Cabon *et al*., 2018; Hernández *et al*., 2022; Sporek and Sporek, 2023). Analysis, monitoring and modeling of ecosystems with more precise, sophisticated and detailed techniques that demand less work to directly measure crown diameter (Pretzsch, 2022). Evaluating the potential density of the target population stand helps to apply the appropriate silvicultural treatments (Yang and Brandeis, 2022). In this way, the timely management of the density of these areas can be decisive in accelerating the growth of

residual trees, which is why mathematical tools are required that relate the size of the trees with their number. In recent decades, different methodologies have been developed to determine the density level of a stand, such as the Reineke density index (Reineke, 1933; Curtis and Reukema, 1970), which is based on the maximum density that a stand can support, and the crown competition factor (CCF) (Krajicek *et al.*, 1961), which helps to determine the number of trees of each diameter category that a hectare can support just at the threshold of the beginning of competition and is estimated from the determination of the maximum crown area projected by trees growing free of competition.

The maximum crown projection area is an important measure for the development of density guides (Rodríguez *et al.*, 2009; Hernández *et al.*, 2013; Bueno *et al.*, 2022)), for the projection of the growth of individual trees as a function of density (Biging and Dobbertin, 1995; Hasenauer *et al.*, 1994; Pretzsch *et al.*, 2022), for the determination of light conditions in the understory that in turn are important for the establishment and development of regeneration (Crookston and Stage , 1999), and for the evaluation of the effect of competition on individual trees (Smith *et al.,* 1992; Corral *et al* ., 2004; Colin *et al.*, 2018; Arnoni *et al.*, 2020). Indeed, crown characteristics are also sometimes used to attribute social ranks to trees in a stand, driving the marking of trees for thinning (Bravo *et al.*, 2020).

For the development of maximum crown diameter models, the crown diameter of a sample of trees growing in open spaces free of competition is usually related to their normal diameter (Bechtold, 2003; Yang and Huang, 2017: Qiu *et al.*, 2023). Other variables such as the geographic location of the trees, elevation, exposure and slope have provided marginal improvements in the estimation of the maximum crown diameter (Paine and Hann, 1982; Hasenauer, 1997). Currently, the use of indicators and statistical models to support forest management practices is increasing (Marchi *et al.*, 2020). Despite the importance of the crown competition factor in the management of forest natural resources, there are currently no equations to estimate the maximum crown size of most commercially important forest species in the state of Durango. For this reason, the objective of this study was to compare different regression equations to predict the maximum crown diameter of *Pinus cooperi* in the forest region of El Salto, Durango.

MATERIALS AND METHODS

Area of study

The study was carried out in the Regional Forest Management Unit 1008 (UMAFOR 1008), which includes the municipality of Pueblo Nuevo and part of the municipality of Durango. The UMAFOR covers approximately 507,127 ha and is located in the mountainous massif of the Sierra Madre Occidental, southwest of the state of Durango (Figure 1). The predominant type of vegetation corresponds to mixed forests with species of the genera *Pinus* and *Quercus* mainly. The height above sea level varies from 2,400 m to 2,600 m. The prevailing climate is temperate semi-cold with an annual precipitation regime that fluctuates from 900 to 1,200 mm and an average annual temperature that varies from 8°C in the highest parts to 24°C in the lowest parts (Instituto Nacional de Estadística Geografía e Informática [INEGI], 2015).
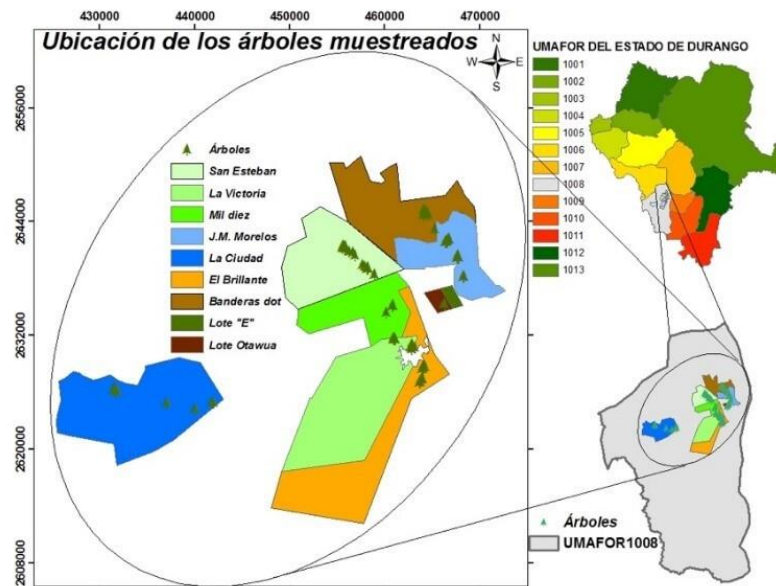

Figure 1. - *Location of sampling plots of Pinus cooperi*

Sampling

The data come from 95 trees randomly selected through targeted sampling. The main characteristic for selecting the sample tree was that it would not be found growing in competition with other trees, nor that there were any stumps close to the target tree within a 30-meter radius, in order to ensure that it developed in a competition-free environment. For each selected tree, the normal diameter (D), total height (HT), clean stem height (ABLC) (insertion of the first whorl with live needles that is part of the crown as a whole), height of the first live needles (ABLF), age in years, crown radius in four directions that coincide with the cardinal points (CR), crown length (CL) were recorded (Figure 2).
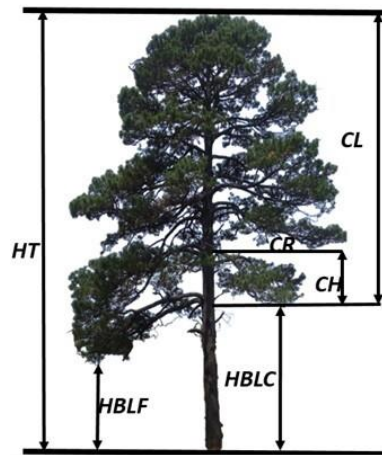

Figure 2.- Main crown variables and geometric relationships used in the development of the maximum crown diameter equation. HBLC: height of the crown base (m); HBLF: height of the first living needles (m); CR: radius of the crown at each measurement point (m); CH: height of the crown from HBLC (m); CL: total length of the crown (m); HT: total height of the tree (m)

Likewise, the name of the property, UTM Datum WGS 84 coordinates, height above sea level, exposure and slope were obtained as control information for each tree. Table 1 presents the most important descriptive statistics for the 95 sample trees.

Table 1. - Descriptive statistics of the sample trees

Variable	Average	Maximum	Minimum	Standard deviation
D	44.97	83.5	23.50	14.56
HT	10.75	20.4	5.60	3.87
HBLC	2.55	8.8	0.07	1.89
HBLF	1.19	4.8	0.07	0.99
HCM	4.43	12.0	0.2	2.44
CL	8.21	14.3	4.2	2.43
DMC	8.99	15.1	4.78	2.58

D: normal diameter (cm), HT: total height (m), HBLC: height of the base of the crown (insertion of the first whorl with living needles that is part of the crown as a whole) (m), HBLF: height of the first living needles (m); HCM: maximum crown height, CL: total crown length (m), and DMC: maximum crown diameter.

Models

Three regression models (simple linear, second-degree polynomial and potential) were fitted to predict maximum crown diameter independently for each of the predictor variables D, HT, HBLC and age using the ordinary least squares (OLS) technique, with the MODEL procedure of the SAS/ETS® program (SAS Institute Inc., 2008). The mathematical expressions of the models analyzed are Equation 1, Equation 2 and Equation 3:

$$dmc = \beta_0 + \beta_1 * Vi \quad (1)$$

$$dmc = \beta_0 + \beta_1 * Vi + \beta_2 * Vi^2 \quad (2)$$

$$dmc = \beta_0 * Vi^{\beta_1} \quad (3)$$

Where: dmc: maximum crown diameter; Vi: Independent variable; βi: are the parameters to be estimated in the adjustment.

Using the best model as a base, the maximum canopy area and percentage of coverage by diameter category were estimated, as well as the number of trees and the basal area sufficient to cover one hectare. The relationship between the number of trees and the average normal diameter allowed the elaboration of a density graph at different percentages of canopy coverage.

Model comparison and selection

The analysis of the adjustment capacity of the equations was based on the graphical analysis of the residues and on the values of two statistics: the coefficient of determination (R^2) and the root mean square error (RMSE), whose mathematical expressions are the following Equation 4 and Equation 5:

$$R^2 = 1 - \frac{\sum_{i=1}^{i=n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{i=n}(Y_i - \overline{Y})^2}(4)$$

$$REMC = \sqrt{\frac{\sum_{i=1}^{i=n}(Y_i - \hat{Y}_i)^2}{n-p}}(5)$$

Where: Y, $\hat{Y}$, $\overline{Y}$ are respectively the observed, estimated and average values of the dependent variable, n is the total number of observations used to fit the model and p is the number of parameters to estimate.

RESULTS AND DISCUSSION

The statistical analysis indicate that the independent variable that presented the best fit in the three models used to estimate the maximum crown diameter of *Pinus cooperi* was the normal diameter (R^2 from 0.81 to 0.83 and RCME from 1.07 to 1.10), followed by total height and age (Table 2). Similar results were obtained by Coombes (2019) where he obtained a higher adjusted R^2 (0.85) using quadratic regression. Crown diameter estimates typically employ regression models that use normal diameter as their main explanatory variable due to its high correlation with crown diameter and easy measurement compared to other

variables such as crown height and length, among others (Quadri, 2019). Studies carried out on a variety of wood species have shown that normal diameter has been a reliable predictor variable in estimating maximum crown diameter (Bechtold, 2003; Condes and Sterba, 2005; Rodríguez *et al.*, 2009; Martin *et al.*, 2012; Chen *et al.*, 2021; Qiu *et al.*, 2022; Qiu *et al.*, 2023). Although other authors have used normal diameter as the main predictor variable, total height has been added to this variable (Li, 2020; Pretzsch *et al.*, 2020), crown length (Moeur, 1981), crown area projected above ground (Jucker *et al.*, 2017; González- Benecke *et al.*, 2022), density (Bragg, 2001; Sporek and Sporek , 2023) exposure, slope and altitude (Curtis and Reukema 1970) and in the use of artificial neural networks to improve the level of prediction (Bueno *et al.*, 2022; Ou and Quiñonez, 2023).

Table 2. *- Fit statistics of the variables to model the maximum crown diameter of Pinus cooperi*

Model	Predictor variable							
	DN		HT		HBLC		AGE	
	R^2	REMC	R^2	REMC	R^2	REMC	R^2	REMC
[1]	0.81	1.10	0.57	1.70	0.27	2.21	0.50	1.82
[2]	0.83	1.07	0.66	1.50	0.38	2.05	0.55	1.73
[3]	0.82	1.09	0.59	1.64	0.33	2.11	0.54	1.75

DN: normal diameter (cm), HT: total height (m), HBLC: height of the base of the crown (insertion of the first whorl with living needles that is part of the crown as a whole) (m), R^2: coefficient of determination, REMC: mean square error

Table 3 shows the parameter estimators of the three models and their fit statistics using normal diameter as the predictive variable. In all cases the parameters were significant at the 5% significance level.

***Table 3**.- Estimated parameters and fit statistics of the models used to model the maximum crown diameter of Pinus cooperi, depending on the maximum crown diameter*

Model	Parameters		Standard error	REMC	R²
[1]	β_0	1,7574	0.4465	1,1092	0.819
	β_1	0.1608	0.0094		
[2]	β_0	-1.1081	1,2764	1,0700	0.831
	β_1	0.2889	0.0544		
	β_2	-0.00129	0.00054		
[3]	β_1	0.4342	0.0805	1,0906	0.824
	β_2	0.7986	0.0472		

R^2: coefficient of determination, REMC: root mean square error.

The Figure 3 shows a homogeneous distribution of the model errors, indicating that there is homoscedasticity, so the assumptions for the regression analysis are fulfilled.
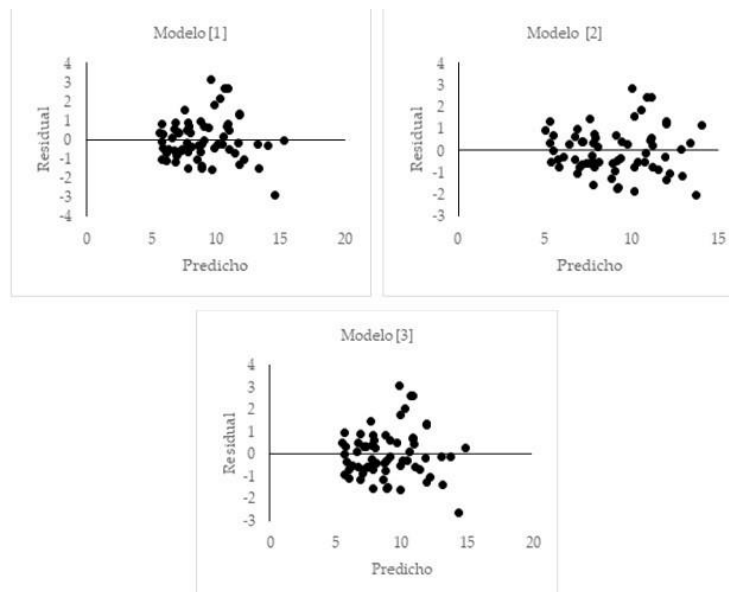

***Figure 3**.- Predicted values versus residuals of the three models analyzed*

On the other hand, Figure 4 shows the graphs of the predicted values against the observed values, observing that the three models provide a good prediction of the maximum crown diameter using the normal diameter as an independent variable.
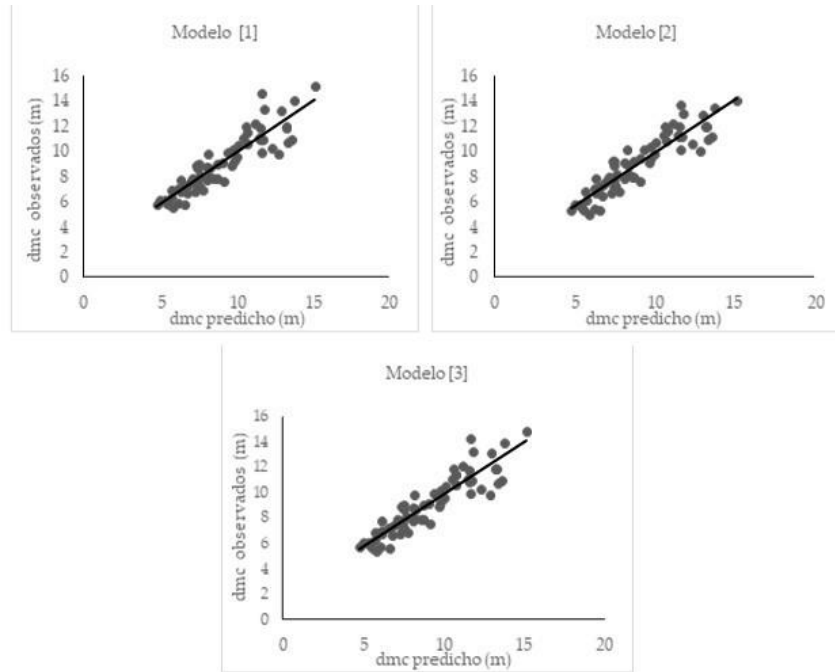
Figure 4. - Maximum observed crown diameters versus predicted values obtained through the use of the three models using the normal diameter as the predictive variable. The solid line represents a linear model fitted to the scatter plot

Although the fit of the three models presents very little variation in the coefficient of determination and in the precision estimator (REMC), the regression equation derived from the second-degree polynomial model provides slightly better predictions which has already been reported in other research (Sharma *et al.*, 2017; Bera *et al.*, 2021), therefore, its use is recommended to estimate the maximum crown diameter of *Pinus cooperi* through the following expression Equation 6:

$$dmc = -1.1081 + 0.2889 * dn - 0.00129 * dn^2 (6)$$

Where:

dmc : maximum crown diameter (m)

dn : normal diameter (m)

The inclusion of dn^2 in the model is justified since much of the existing literature to predict crown diameter via normal diameter (Bechtold, 2003; Hasenauer, 1997; Lhotka and Loewenstein, 2008; Martin *et al.*, 2012; Chen *et al.*, 2021), as well as previously published maximum crown diameter equations have used dn^2 (Paine and Hann 1982; Smith *et al.*, 1992; Sporek and Sporek, 2023).

With the values of the estimators and multiplying by the ratio, $\frac{\pi}{4}$ we have the equation for the maximum crown area (Amc) (Hutch *et al.*, 1993; Di Salvatore *et al.*, 2021) Equation 7:

$$Amc = \frac{\pi}{4}(-1.1081 + 0.2889 * dn - 0.00129 * dn^2)^2 (7)$$

Where:

Amc : maximum crown area (m^2)

Amc equation for a range of trees with a diameter of 20 to 80 cm, the estimated maximum crown area for *Pinus cooperi* varies from 13.55 m^2 (0.136% cover) to 156.77 m^2 (1.568%) cover. In turn, the minimum average number of trees necessary to cover 100% of the surface of a hectare by diameter category within the interval of 20 to 85 cm varied from 738 (23.18 m^2 ha^{-1}) to 64 (36.20 m^2 ha^{-1}), respectively (Table 4). This number of trees is slightly lower than those estimated with the equation reported by Quiñones and Ramírez (1998) for the same species that is present in a region neighboring the one studied. For *Pinus cooperi*, the average number of trees required to cover 100% of the surface of a hectare by diameter category within the interval of 20 to 85 cm varied from 738 (23.18 m^2 ha^{-1}) to 64 (36.20 m^2 ha^{-1}), respectively (Table 4). This number of trees is *slightly* lower than those estimated with the equation reported by Quiñones and Ramírez (1998) for the same species that is present in a region neighboring the one studied. To *Pinus rudis* Endl. in Oaxaca, similar results were obtained in the diameter category of 20 with 740 trees (Martínez *et al.*, 2021). According to this equation, between 627 and 80 trees per hectare are estimated within the range of the diameter categories studied. Hernández *et al.* (2013) report that in *Pinus teocote* Schlecht. Et Cham. from the state of Hidalgo the number of trees decreases from 580 to 55 within the range of 20 to 85 cm of normal diameter, while Rodriguez *et al.* (2009) mentions that in *Pinus*

montezumae Lamb. these decrease from 557 to 168 within a range of 20 to 40 cm of normal diameter.

Table 4. - Density attributes from the estimation of the maximum crown area of *Pinus cooperi*

Dn (cm)	AMC (m^2)	Arbha $^{-1}$	ABha $^{-1}$ (m ^{2} ha $_{-1}$)	ACi (%)
20	13.55	738	23,18	0.136
25	22,13	452	22,18	0.221
30	32,15	311	21,99	0,321
35	43,28	231	22,23	0,433
40	55,21	181	22,76	0,552
45	67,64	148	23,51	0,676
50	80,31	125	24,45	0,803
55	92,96	108	25,56	0,930
60	105,35	95	26,84	1,054
65	117,29	85	28,29	1,173
70	128,56	78	29,94	1,286
75	139,00	72	31,78	1,390
80	148,44	67	33,86	1,484
85	156,77	64	36,20	1,568

Dn : normal diameter, AMC: maximum crown area, Arbha $^{-1}$: trees per hectare, Abha $^{-1}$: basal area per hectare, ACi : percentage cover

The density graph created from the CCF (Figure 5) indicates the threshold of the minimum number of trees per diameter category that is necessary to cover 100% of the surface. From the point of view of density management for timber production purposes, the 100% CCF line is the basis for estimating the level of competition between individuals and optimal land use. A CCF less than 100% indicates the absence of competition and deficiency in land use, so to optimize the quality and quantity of timber production it is desirable to maintain the density of a forest above 100% coverage.
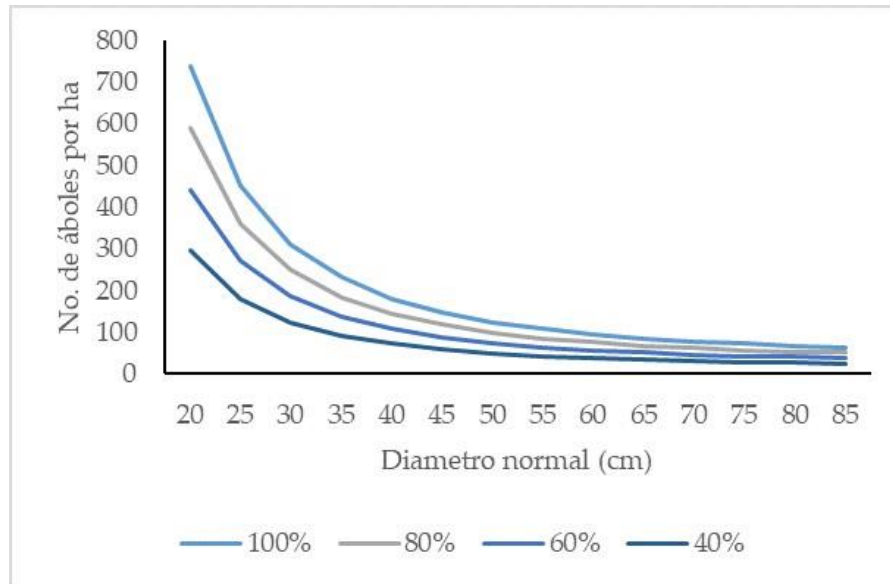
Figure 5.- *Relationship between the number of trees per diameter category at different levels of crown competition factor (CCF)*

CONCLUSIONS

Normal diameter is the most explanatory variable in a simple linear model and a quadratic one and best predictor for maximum crown diameter in *Pinus cooperi* .

The quadratic polynomial model best describes the normal diameter-maximum crown diameter relationship, so it is recommended for use in the study of growth and competition of this species in even-aged and mixed and irregular stands in the state of Durango.

Two forest productivity scenarios can be identified related to the crown competition factor; when it is 100% or higher, timber productivity is optimal; and if it is less than 100%, productivity is not optimal.

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REFERENCES

ARNONI, E., GUIMARÃES, C.A., SCHNEIDER, P.R., HESS, A., LIESENBERG, V., y TAGLIAPIETRA, C., 2020. Modelado de índices de competencia para Araucaria angustifolia en dos sitios en el sur de Brasil. Bosque (Valdivia), [En línea] Vol. 41, no. 1 [Consulta: 1 de septiembre de 2024]. ISSN 0717-9200. DOI: 10.4067/s0717-92002020000100065. Disponible en: http://dx.doi.org/10.4067/S0717-92002020000100065

BERA, D., CHATTERJEE, N., y BERA, S., 2021. Comparative performance of linear regression, polynomial regression and generalized additive model for canopy cover estimation in the dry deciduous forest of West Bengal. Remote Sensing Applications: Society and Environment [En línea]. Vol. 22 [Consulta: 18 de agosto de 2024]. ISSN 2352-9385. DOI: 10.1016/j.rsase.2021.100502. Disponible en: https://doi.org/10.1016/j.rsase.2021.100502.

BECHTOLD, W.A., 2003. Crown-Diameter Prediction Models for 87 Species of Stand-Grown Trees in the Eastern United States. Southern Journal of Applied Forestry, [En línea]. Vol. 27, n.º 4 [Consulta: 8 septiembre de 2024]. ISSN 1938-3754. DOI: 10.1093/sjaf/27.4.269. Disponible en: https://doi.org/10.1093/sjaf/27.4.269.

BIGING, G., y DOBBERTIN, M., 1995. Evaluation of competition indices in individual tree growth models. Forest Science [En línea]. Vol. 41, n.°2 [Consulta: 18 de septiembre de 2024. ISSN 1938-3738. DOI: 10.1093/forestscience/41.2.360. Disponible en: https://doi.org/10.1093/forestscience/41.2.360

BRAGG, D. C., 2001. A local basal area adjustment for crown width prediction. Northern Journal of Applied Forestry [En Linea]. Vol. 18, n.° 1. [Consulta: 25 de Agosto de 2024]. ISSN 1938-3762. DOI: 10.1093/njaf/18.1.22. Disponible en: https://doi.org/10.1093/njaf/18.1.22

BRAVO, A., MARCHI, M., TRAVAGLINI, D., PELLERI, F., MANETTI, M.C., CORONA, P., et al., 2020. Adoption of new silvicultural methods in Mediterranean forests: the influence of educational background and sociodemographic factors on marker decisions. Annals of Forest Science [En línea], vol. 77, n.º 2 [Consulta: el 21 de septiembre de 2024]. ISSN 1297-966X. DOI: 10.1007/s13595-020-00947-z. Disponible en: https://doi.org/10.1007/s13595-020-00947-z.

BUENO, G.F., COSTA, E.A., FINGER, C.A.G., LIESENBERG, V., y BISPO, P.D.C., 2022). Machine learning: crown diameter predictive modeling for open-grown trees in the cerrado biome, Brazil. Forests [En línea], Vol. 13, n.° 8 [Consulta: el 12 de agosto de 2024]. ISSN 1999-4907. DOI: 10.3390/f13081295. Disponible en: https://doi.org/10.3390/f13081295

CABON, A., MOUILLOT, F., LEMPEREUR, M., OURCIVAL, J.M., SIMIONI, G., y LIMOUSIN, J.M., 2018. Thinning increases tree growth by delaying drought-induced growth cessation in a Mediterranean evergreen oak coppice. Forest Ecology and Management [En línea]. Vol. 409 [Consulta: 9 de septiembre de 2024] ISSN 0378-1127. DOI: 10.1016/j.foreco.2017.11.030. Disponible en: https://doi.org/10.1016/j.foreco.2017.11.030.

CHEN, Q., DUAN G., LIU Q., YE Q., SHARMA R.P.., CHE Y, LIU H., y FU, L., 2021. Estimating crown width in degraded forest: A two-level nonlinear mixed-effects crown width model for Dacrydium pierrei and Podocarpus imbricatus in tropical China. Forest Ecology and Management [En línea]. Vol. 497 [Consulta: 20 de septiembre de 2024]. ISSN 1999-4907. DOI: 10.3390/f13081295. Disponible en: https://doi.org/10.3390/f13081295

CISNEROS, A.B., MOGLIA G., y ÁLVAREZ, J.A., 2019. Morfometría de copa en Prosopis alba Griseb. Ciência Florestal [En linea]. Vol. 29, n.°2. [Consulta: 29 de Agosto de 2024]. ISSN 1980-5098. DOI: 10.5902/1980509826846. Disponible en: https://doi.org/10.5902/1980509826846.

COLÍN, J.G., AGUIRRE, O.A., CORRAL, J.J., VIVEROS, E., CORRAL, S., y CRECENTE, F., 2018. Influence of competition on the diametric growth of Pinus durangensis Martínez in Durango, Mexico. Revista Mexicana de Ciencias Forestales [En línea]. Vol. 9, n.°45 [Consulta: 10 de julio de 2024]. ISSN 2448-6671. DOI: 10.29298/rmcf.v9i45.145 Disponible en: https://doi.org/10.29298/rmcf.v9i45.145

CONDES, S., y STERBA, H., 2005. Derivation of compatible crown width equations for some important tree species of Spain. Forest Ecology Management [En línea]. Vol. 217 n.°2 [Consulta: 28 de julio de 2024]. ISSN 1999-4907. Disponible en: 203–218. https://doi.org/10.1016/j.foreco.2005.06.002

COOMBES, A., MARTIN, J., y SLATER, D., 2019. Defining the allometry of stem and crown diameter of urban trees. Urban Forestry & Urban Greening [En línea]. Vol. 44. [Consulta: 17 de septiembre de 2024]. ISSN 1618-8667. DOI: 10.1016/j.ufug.2019.126421. Disponible en; https://doi.org/10.1016/j.ufug.2019.126421

CORRAL, J.J., ÁLVAREZ, J.G., RUÍZ, A.D., y GADOW, K., 2004. Compatible height and site index models for five pine species in El Salto, Durango (Mexico). Forest Ecology and Management [En línea]. Vol. 201. [Consulta: 8 de julio de 2024]. ISSN 1999-4907. DOI: 10.1016/j.foreco.2004.05.060. Disponible en: https://doi.org/10.1016/j.foreco.2004.05.060

CROOKSTON, N.L., y STAGE, A.R., 1999. Percent canopy cover and stand structure statistics from the Forest Vegetation Simulator. US Department of Agriculture, Forest Service, Rocky Mountain Research Station: Ogden, UT, USA, 1999. 63 p. Disponible en https://www.fs.usda.gov/fmsc/ftp/fvs/docs/gtr/percancv.pdf

CURTIS, R.O., y REUKEMA, L., 1970. Crown development and site estimates in a Douglas fir plantation spacing test. Forest Science [En linea]. Vol. 16, n.°3 [12 de julio de 2024]. ISSN 1938-3738. DOI: 10.1093/forestscience/16.3.287. DOI: 10.1093/forestscience/16.3.287. Disponible en; https://doi.org/10.1093/forestscience/16.3.287

DI SALVATORE, U., MARCHI, M., y CANTIANI, P., 2021. Single-tree crown shape and crown volume models for Pinus nigra JF Arnold in central Italy. Annals of Forest Science [En línea]. Vol. 78 [Consultado 3 de septiembre de 2024]. ISSN 1297-966X. DOI: 10.1007/s13595-021-01099-4. Disponible en: https://doi.org/10.1007/s13595-021-01099-4

GIVNISH, T.J., 2020. The adaptive geometry of trees. The American Naturalist [En línea]. Vol. 195. N.°6 [Consulta: 11 de septiembre de 2024]. ISSN 1537-5323. DOI: 10.1086/708498. Disponible en: https://doi.org/10.1086/708498.

GONZALEZ-BENECKE C.A., FERNÁNDEZ, M.P., GAYOSO, J., PINCHEIRA, M., y WIGHTMAN, M., 2022. Using tree height, crown area and stand-level parameters to estimate tree diameter, volume, and biomass of Pinus radiata, eucalyptus globulus and eucalyptus nitens. Forests [En línea]. Vol. 13, n.°12 [Consulta: 29 de agosto de 2024]. ISSN 1999-4907. .DOI: 10.3390/f13122043 Disponible en: https://doi.org/10.3390/f13122043

HASENAUER, H. (1997). Dimensional relationships of open-grown trees in Austria. Forest Ecology and Management [En línea]. Vol. 96, n.° 3 [Consulta: 18 de agosto de 2024]. ISSN 1999-4907. DOI: 10.1016/S0378-1127(97)00057-1. Disponible en: https://doi.org/10.1016/S0378-1127(97)00057-1

HASENAUER, H., MOSER, M., y ECKMU¨LLNER, O., 1994. MOSES — a computer simulation program for modelling stand response. In: Pinto da Costa, M.E., Preuhsler, T. (Eds.), Mixed Stands, Research Plots, Measurement and Results, Models. Inst. Superior de Agronomia, University Technica de Lisboa, Lisboa Codex, Portugal (Appendix 1:1).

HERNÁNDEZ, J., GARCÍA, J.J., MUÑOS, H.J., GARCÍA, X., SÁENS, T., FLORES, C., y HÉRNÁNDEZ, A., 2013. Guía de densidad para manejo de bosques naturales de Pinus teocote Schlecht. Et Cham. En Hidalgo. Revista Mexicana de Ciencias Forestales, [En línea]. Vol. 4, n.° 19 [Consulta: 19 de agosto de 2024]. ISSN 2448-6671. Disponible en:

https://www.scielo.org.mx/scielo.php?pid=S2007-11322013000500006&script=sci_arttext

HERNÁNDEZ, J., REYES, V.J., y BELTRÁN, L., 2022. La copa como indicador fotosintético relevante en el manejo forestal de bosques templados. Revista Mexicana de Ciencias Forestales [En linea]. Vol 13, n.° 74 [Consulta: 10 de septiembre de 2024]. ISSN 2448-6671. DOI: 10.29298/rmcf.v13i74.1267. Disponible en: https://doi.org/10.29298/rmcf.v13i74.1267

HESS, A.F., LOIOLA, T., ARRUDA DE SOUZA, I., y NASCIMENTO. B., 2016. Morfometría de la copa de Araucaria angustifolia en sitios naturales en el sur de Brasil. Bosque (Valdivia) [En línea]. Vol. 37, n.° 3 [Consulta 20 de Agosto de 2024]. ISSN 0717-9200. DOI: 10.4067/S0717-92002016000300017. Disponible en; http://dx.doi.org/10.4067/S0717-92002016000300017

HUSCH, B., MILLER, C., y BEERS, T., 1993. Forest mensuration. Krieger Publishing Co. Malabar, Florida, USA, 410 p. Disponible en: https://books.google.es/books?hl=es&lr=&id=p0v3m8Pau-kC&oi=fnd&pg=PR11&dq=Forest+mensuration&ots=LJFOmoHGVu&sig=6fep_KKHmxk2UCe_poUIlw9-yDc#v=onepage&q=Forest%20mensuration&f=false

INSTITUTO NACIONAL DE ESTADÍSTICA GEOGRAFÍA E INFORMÁTICA (INEGI), 2015. Anuario Estadístico Durango [En línea]. [consulta: 26 de agosto 2024]. Disponible en: http://www.datatur.sectur.gob.mx/ITxEF_Docs/DGO_ANUARIO_PDF15.pdf

JUCKER, T., CASPERSEN, J., CHAVE, J., ANTIN, C., BARBIER, N., BONGERS, F., et al., 2017. Allometric equations for integrating remote sensing imagery into forest monitoring programmes. Global change biology [En línea]. Vol. 23, n.°1 [Consulta: 19 de Agosto de 2024]. ISSN 1354-1013. DOI: 10.1111/gcb.13388 Disponible en: https://doi.org/10.1111/gcb.13388

KRAJICEK, J., BRINKMAN, K., y GINGRICH, S., 1961. Crown competition—a measure of density. Forest Science [En línea]. Vol. 7 [Consulta: 8 de septiembre de 2024]. ISSN 1938-3738. DOI: 10.1093/forestscience/7.1.35. Disponible en: https://doi.org/10.1093/forestscience/7.1.35

LHOTKA, J.M., y LOEWENSTEIN, E.F., 2008. An Examination of Species- Specific Growing Space Utilization. Canadian Journal of Forest Research [En línea]. Vol. 38, n.° 3 [Consulta: 11 de gusto de 2024]. ISSN 1208-6037. DOI: 10.1139/X07-147. Disponible en: https://doi.org/10.1139/x07-147.

LI, Y., WANG, W., ZENG, W., WANG, J., y MENG, J., (2020. Development of Crown Ratio and Height to Crown Base Models for Masson Pine in Southern China. Forests [En línea]. Vol. 11, n.° 11 [Consulta 23 de agosto de 2024]. ISSN 1999-4907. DOI: 10.3390/f11111216. Disponible en: https://doi.org/10.3390/f11111216

MARCHI, M., SCOTTI, R., RINALDINI, G., y CANTIANI, P., 2020. Taper Function for Pinus nigra in Central Italy: Is a More Complex Computational System Required? Forests [En línea]. Vol. 11, n.º 4 [Consulta: 18 de Agosto de 2024]. ISSN 1999-4907. DOI: 10.3390/f11040405. Disponible en: https://doi.org/10.3390/f11040405.

MARTIN, N.A., CHAPPELKA, A., LOEWENSTEIN, E.F., KEEVER, G.J., y SOMERS, G., 2012. Predictive open-grown crown width equations for three oak species Planted in a Southern Urban Locale. Arboriculture & Urban Forestry [En línea]. Vol. 38, n.° 2 [Consulta: 29 de julio de 2024]. ISSN 2155-0778. DOI: 0.48044/jauf.2012.010. Disponible en: https://doi.org/10.48044/jauf.2012.010

MARTÍNEZ, D., CABALLERO, P., FILIO, E., GARZÓN, A., ORTIZ, R., CRUZ, O.L., APARICIO, C., y SANDOVAL, C., 2021. Guía de densidad para el manejo de rodales naturales de Pinus rudis Endl. en Oaxaca. Revista Mexicana de Ciencias Forestales [En línea]. Vol. 12, n.° 64 [Consulta 17 de septiembre de 2024]. ISSN 2448-6671. DOI: 10.29298/rmcf.v12i64.822. Disponible en linea: https://doi.org/10.29298/rmcf.v12i64.822

MOEUR, M., 1981. Crown width and foliage weight of northern Rocky Mountain conifers. [En linea]. USDA Forest Service Research Paper, INT-283, 14p. [Consulta: 1 de agosto de 2024]. Disponible en: https://books.google.es/books?hl=es&lr=&id=PBGOcgtHiaYC&oi=fnd&pg=PA1&dq=Crown+width+and+foliage+weight+of+northern+Rocky+Mountain+conifers.+USDA+Forest+Service+Research+Paper,+&ots=CnUq0F6PM3&sig=tGdl2DwEN9Txy3lQ2BxWQMUX2jQ#v=onepage&q=Crown%20width%20and%20foliage%20weight%20of%20northern%20Rocky%20Mountain%20conifers.%20USDA%20Forest%20Service%20Research%20Paper%2C&f=false

OU, Y., y QUIÑÓNEZ, G., 2023. Modeling Height–Diameter Relationship Using Artificial Neural Networks for Durango Pine (Pinus durangensis Martínez) Species in Mexico. Forests [En línea]. Vol. 14, n.° 8 [Consulta: 12 de septiembre de 2024]. ISSN 1999-4907. DOI: 10.3390/f14081544. Disponible en: https://doi.org/10.3390/f14081544

PAINE, D.P., y HANN, D.W., 1982. Maximum crown-width equations for southwestern Oregon tree species [En línea]. Forest Research Laboratory, Oregon State University, Corvallis. Research Paper. 46, 20 p. [Consulta: 12 de agosto de 2024]. Disponible en: https://ir.library.oregonstate.edu/downloads/b2773w92s

PRETZSCH, H., AHMED, S., JACOBS, M., SCHMIED, G., y HILMERS, T., 2022. Linking crown structure with tree ring pattern: methodological considerations and proof of concept. Trees [En línea]. Vol. 36, n.° 4 [Consulta: 9 de septiembre de 2024]. ISSN 1432-2285. DOI: 10.1007/s00468-022-02297-x. Disponible en: https://doi.org/10.1007/s00468-022-02297-x

PRETZSCH, H., BIBER, P., y DURSKY, J., 2002. The single tree-based stand simulator SILVA: Construction, application and evaluation. Forest Ecology Management [En linea]. Vol. 162, n.°1 [Consulta: 13 de agosto de 2024]. ISSN 1999-4907. DOI: Disponible en: https://doi.org/10.1016/S0378-1127(02)00047-6

QUADRI, O.A., 2019. Development and evaluation of linear and non-linear models for diameter at breast height and crown diameter of TriplochitonScleroxylon (K. Schum) Plantations in Oyo State, Nigeria. Journal of Agriculture and Veterinary Science [En línea]. Vol. 12, n.° 6 [Consulta: 12 de septiembre de 2024]. ISNN 2319-2380. DOI: 10.9790/2380-1206014752. Disponible en: https://doi.org/10.9790/2380-1206014752

QUIÑONES, A., y RAMÍREZ, H., 1998. Evaluación de la calidad de sitio y del efecto de la densidad en bosques de Pinus cooperi del estado de Durango. Revista Chapingo Serie Ciencias Forestales y del Ambiente. Vol. 4, n.° 2 [Consulta: 10 de agosto de 2024]. ISSN 2007-4018.

QIU, S., LIANG, R., WANG, Y., LUO, M., y SUN, Y., 2022. Comparative analysis of machine learning algorithms and statistical models for predicting crown width of Larix olgensis. Earth Science Informatics [En línea] Vol. 15 n.° 4 [Consulta: 10 de septiembre de 2024]. ISSN 1865-0481. DOI: Disponible en: 10.1007/s12145-022-00854-z https://doi.org/10.1007/s12145-022-00854-z

REINEKE, L.H., 1933. Perfecting a stand density index for even aged forest. Journal of Agricultura Research [En línea] Vol. 46, n.° [Consulta: 18 de Agosto de 2024]. ISSN 0095-9758. Disponible en: https://cmapspublic3.ihmc.us/rid=1N4TSLMJK-JNTN9D-14QK/Perfecting%20a%20stand-density%20index%20for%20even-aged%20forests.PDF

RODRÍGUEZ, L.R., RAZO, R., DÍAZ, H., y MEZA, J., 2009. Guía de densidad para Pinus montezumae en su área de distribución natural en el Estado de Hidalgo. [Consulta: 15 de septiembre de 2024]. Folleto Técnico No. 1. Universidad Autónoma del Estado de Hidalgo. Instituto de Ciencias Agropecuarias. Área Académica de Ingeniería Forestal. Tulancingo de Bravo, Hgo. México. 31 p. Disponible en: https://www.uaeh.edu.mx/investigacion/icap/LI_IntGenAmb/Rodri_Laguna/1.pdf

SANTIAGO-GARCÍA, W., DE LOS SANTOS, P.H.M., ÁNGELES G., VALDEZ J.R., DEL VALLE, D.H., y CORRAL, J.J. (2013). Auto-aclareo y guías de densidad para Pinus patula mediante el enfoque de regresión de frontera estocástica. Agrociencia [En linea]. Vol. 47, n.° 1 [Consulta: 29 de agosto de 2024]. ISNN 2521-9766. Disponible en: https://www.scielo.org.mx/scielo.php?pid=S1405-31952013000100007&script=sci_arttext

SECRETARÍA DEL MEDIO AMBIENTE Y RECURSOS NATURALES (SEMARNAT), 2019. Producción forestal maderable y no maderable. [En línea]. [consulta: 16 de agosto 2024]. Disponible en: https://apps1.semarnat.gob.mx:445/dgeia/indicadores17/conjuntob/indicador/07_f orestales/7_2.html

SHARMA, R.P., BÍLEK, L., VACEK, Z., y VACEK, S., 2017. Modelling crown width–diameter relationship for Scots pine in the central Europe. Trees [En línea]. Vol. 31 [Consulta: 28 de agosto de 2024]. ISSN 1432-2285. DOI: 10.1007/s00468-017-1593-8, Disponible en; https://doi.org/10.1007/s00468-017-1593-8

SPOREK, M., y SPOREK, K., 2023. Allometric Model of Crown Length for Pinus sylvestris L. Stands in South-Western Poland. Forests [En linea]. Vol. 14, n.° 9 [Consulta: 16 de agosto de 2024] ISSN 1999-4907. DOI: 10.3390/f14091779. Disponible en: https://doi.org/10.3390/f14091779

SMITH, W.R., FARRAR, J., y MURPHY, P.A., 1992. Crown and basal area relationships of open-grown southern pines for modelling competition and growth. Canadian Journal of Forest Research [En línea]. Vol. 22, n.° 3 [Consulta: 10 de se´pteiembre de 2024]. ISSN 1208-6037, DOI: 10.1139/x92-044. Disponible en: https://doi.org/10.1139/x92-044

SOTO, J.A., LÓPEZ, C.A., CORRAL, J.J., WEHENKEL, C., ÁLVAREZ, J.G., y CRECENTE, F. 2016. Desarrollo de un modelo de perfil de copa para Pinus cooperi Blanco en la UMAFOR 1008, Durango, México. Revista Chapingo Serie Ciencias Forestales y del Ambiente [En linea]. Vol. 22, n.° 2 [Consulta6 de septiembre de 2024]. ISSN ISSN 2007-

4018. DOI: 10.5154/r.rchscfa.2015.09.040. Disponible en: https://doi.org/10.5154/r.rchscfa.2015.09.040

TAMARIT, U.J.C., QUIÑONEZ, B.G., y HERNÁNDEZ, R.J., 2020. Aspectos metodológicos para generar diagramas de manejo de la densidad de rodales con base en el índice de Reineke. Revista Mexicana de Ciencias Forestales [En línea]. Vol 11, n.° 61 [Consulta 10 de septiembre de 2024].ISSN 2448-6671. DOI: 10.29298/rmcf.v11i61.728. Disponible en: https://doi.org/10.29298/rmcf.v11i61.728

VOSPERNIK, S., y STERBA, H., 2015. Do competition-density rule and self-thinning rule agree?. Annals of Forest Science [En línea]. Vol. 72 [Consulta: 10 de septiembre de 2024]. ISSN 1297-966X. DOI: 10.1007/s13595-014-0433-x Disponible en; https://doi.org/10.1007/s13595-014-0433-x

YANG, S.I., y BRANDEIS, T.J., 2022. Estimating maximum stand density for mixed-hardwood forests among various phy siographic zones in the eastern US. Forest Ecology and Management [En línea]. Vol. 521 [Consulta: 18 de septiembre de 2024]. ISSN 1999-4907. DOI: Disponible en: https://doi.org/10.1016/j.foreco.2022.120420

YANG, Y., y HUANG, S. 2017. Allometric modelling of crown width for white spruce by fixed-and mixed-effects models. The Forestry Chronicle [En línea]. Vol. 93, n.° 2 [6 de septiembre de 2024]. ISSN 1499-9315. DOI: 10.5558/tfc2017-020. Disponible en: https://doi.org/10.5558/tfc2017-020

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The authors declare not to have any interest conflicts.

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The authors have participated in the writing of the work and analysis of the documents.